## Maple 2018.2 Integration Test Results on the problems in "1 Algebraic functions/1.2 Trinomial products/1.2.4 Improper"

Test results for the 40 problems in "1.2.4.2 (d x)<sup>m</sup> (a  $x^q+b x^n+c x^(2 n-q))^p.txt$ "

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \frac{x^8}{\left(cx^4 + bx^3 + ax^2\right)^2} \, \mathrm{d}x$$

Optimal(type 3, 146 leaves, 8 steps):

$$\frac{2(-3\,a\,c+b^2)\,x}{c^2(-4\,a\,c+b^2)} - \frac{b\,x^2}{c\,(-4\,a\,c+b^2)} + \frac{x^3(b\,x+2\,a)}{(-4\,a\,c+b^2)(c\,x^2+b\,x+a)} - \frac{2(6\,a^2\,c^2-6\,a\,b^2\,c+b^4)\operatorname{arctanh}\left(\frac{2\,cx+b}{\sqrt{-4\,a\,c+b^2}}\right)}{c^3(-4\,a\,c+b^2)^{3/2}} - \frac{b\ln(c\,x^2+b\,x+a)}{c^3}$$

Result(type 3, 568 leaves):

$$\frac{x}{c^{2}} + \frac{2xa^{2}}{c(cx^{2}+bx+a)(4ac-b^{2})} - \frac{4xab^{2}}{c^{2}(cx^{2}+bx+a)(4ac-b^{2})} + \frac{xb^{4}}{c^{3}(cx^{2}+bx+a)(4ac-b^{2})} - \frac{3ba^{2}}{c^{2}(cx^{2}+bx+a)(4ac-b^{2})} + \frac{b^{3}a}{c^{3}(cx^{2}+bx+a)(4ac-b^{2})} - \frac{4\ln((4ac-b^{2})(cx^{2}+bx+a))ab}{c^{2}(4ac-b^{2})} + \frac{\ln((4ac-b^{2})(cx^{2}+bx+a))b^{3}}{c^{3}(4ac-b^{2})} - \frac{4\ln((4ac-b^{2})b)}{\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}} + \frac{12\arctan\left(\frac{2(4ac-b^{2})cx+(4ac-b^{2})b}{\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}}\right)a^{2}}{c^{2}\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}} + \frac{2\arctan\left(\frac{2(4ac-b^{2})cx+(4ac-b^{2})b}{\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}}\right)b^{2}a}{c^{2}\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}} + \frac{2\arctan\left(\frac{2(4ac-b^{2})cx+(4ac-b^{2})b}{\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}}\right)b^{2}a}{c^{2}\sqrt{64a^{3}c^{3}-48a^{2}b^{2}c^{2}+12ab^{4}c-b^{6}}}}$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{cx^4 + bx^3 + ax^2}}{x^4} \, \mathrm{d}x$$

Optimal(type 3, 96 leaves, 5 steps):

$$\frac{(-4\,a\,c+b^2)\operatorname{arctanh}\left(\frac{x\,(b\,x+2\,a)}{2\sqrt{a}\,\sqrt{c\,x^4+b\,x^3+a\,x^2}}\right)}{8\,a^{3/2}} - \frac{\sqrt{c\,x^4+b\,x^3+a\,x^2}}{2\,x^3} - \frac{b\sqrt{c\,x^4+b\,x^3+a\,x^2}}{4\,a\,x^2}$$

Result(type 3, 206 leaves):

$$-\frac{1}{8x^{3}\sqrt{cx^{2}+bx+a}a^{2}}\left(\sqrt{cx^{4}+bx^{3}+ax^{2}}\left(4a^{3}\sqrt{c}c\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^{2}+bx+a}}{x}\right)x^{2}+2c\sqrt{cx^{2}+bx+a}x^{3}b-\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^{2}+bx+a}}{x}\right)x^{2}b^{2}-4c\sqrt{cx^{2}+bx+a}x^{2}a-2(cx^{2}+bx+a)^{3}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}b^{2}+4(cx^{2}+bx+a)^{3}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}b^{2}+4(cx^{2}+bx+a)^{3}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}b^{2}+4(cx^{2}+bx+a)^{3}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}b^{2}+4(cx^{2}+bx+a)^{3}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}b^{2}+4(cx^{2}+bx+a)^{3}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}+a^{2}xb+2\sqrt{cx^{2}+bx+a}x^{2}+a^{2}}+a^{2}\sqrt{2}xb+2\sqrt{cx^{2}+bx+a}+a^{2}}+a^{$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \frac{\left(cx^4 + bx^3 + ax^2\right)^{3/2}}{x^8} \, \mathrm{d}x$$

Optimal(type 3, 171 leaves, 7 steps):

$$-\frac{(cx^{4}+bx^{3}+ax^{2})^{3/2}}{4x^{7}} - \frac{3(-4ac+b^{2})^{2}\operatorname{arctanh}\left(\frac{x(bx+2a)}{2\sqrt{a}\sqrt{cx^{4}+bx^{3}+ax^{2}}}\right)}{128a^{5/2}} - \frac{(-12ac+b^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{32ax^{3}} + \frac{b(-20ac+3b^{2})\sqrt{cx^{4}+bx^{3}+ax^{2}}}{64a^{2}x^{2}} - \frac{(6cx+b)\sqrt{cx^{4}+bx^{3}+ax^{2}}}{8x^{4}}$$

Result(type 3, 500 leaves):

$$-\frac{1}{128 x^{7} (cx^{2}+bx+a)^{3/2} a^{4}} \left( (cx^{4}+bx^{3}+ax^{2})^{3/2} \left( 48 a^{7/2} c^{2} \ln \left( \frac{2a+bx+2\sqrt{a}\sqrt{cx^{2}+bx+a}}{x} \right) x^{4} - 24 a^{5/2} c \ln \left( \frac{2a+bx+2\sqrt{a}\sqrt{cx^{2}+bx+a}}{x} \right) x^{4} b^{2} + 24 c^{2} (cx^{2}+bx+a)^{3/2} x^{5} a b - 16 c^{2} (cx^{2}+bx+a)^{3/2} x^{4} a^{2} + 24 c^{2} \sqrt{cx^{2}+bx+a} x^{5} a^{2} b - 2 c (cx^{2}+bx+a)^{3/2} x^{5} b^{3} + 3 a^{3/2} \ln \left( \frac{2a+bx+2\sqrt{a}\sqrt{cx^{2}+bx+a}}{x} \right) x^{4} b^{4} - 48 c^{2} \sqrt{cx^{2}+bx+a} x^{4} a^{3} - 24 c (cx^{2}+bx+a)^{5/2} x^{3} a b + 20 c (cx^{2}+bx+a)^{3/2} x^{4} a b^{2} - 6 c \sqrt{cx^{2}+bx+a} x^{5} a b^{3} + 16 c (cx^{2}+bx+a)^{5/2} x^{2} a^{2} + 36 c \sqrt{cx^{2}+bx+a} x^{4} a^{2} b^{2} + 2 (cx^{2}+bx+a)^{5/2} x^{3} b^{3} - 2 (cx^{2}+bx+a)^{3/2} x^{4} b^{4} + 4 (cx^{2}+bx+a)^{5/2} x^{2} a b^{2} - 6 \sqrt{cx^{2}+bx+a} x^{4} a b^{4} - 16 (cx^{2}+bx+a)^{5/2} x a^{2} b + 32 (cx^{2}+bx+a)^{5/2} a^{3} \right) \right)$$

Problem 21: Result more than twice size of optimal antiderivative.

$$\int x^m \left( c \, x^5 + b \, x^3 + a \, x \right) \, \mathrm{d}x$$

Optimal(type 3, 37 leaves, 2 steps):

$$\frac{a x^{2+m}}{2+m} + \frac{b x^{4+m}}{4+m} + \frac{c x^{6+m}}{6+m}$$

Result(type 3, 76 leaves):

$$\frac{x^{2+m} \left(c \, m^2 \, x^4 + 6 \, c \, m \, x^4 + b \, m^2 \, x^2 + 8 \, c \, x^4 + 8 \, b \, m \, x^2 + a \, m^2 + 12 \, b \, x^2 + 10 \, a \, m + 24 \, a\right)}{(6+m) \, (4+m) \, (2+m)}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \frac{x^5}{cx^5 + bx^3 + ax} \, \mathrm{d}x$$

Optimal(type 3, 144 leaves, 5 steps):

$$\frac{x}{c} - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b - \sqrt{-4}ac + b^2}}\right) \left(b + \frac{2ac - b^2}{\sqrt{-4}ac + b^2}\right)\sqrt{2}}{2c^{3/2}\sqrt{b - \sqrt{-4}ac + b^2}} - \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b + \sqrt{-4}ac + b^2}}\right) \left(b + \frac{-2ac + b^2}{\sqrt{-4}ac + b^2}\right)\sqrt{2}}{2c^{3/2}\sqrt{b + \sqrt{-4}ac + b^2}}$$

Result(type 3, 342 leaves):

$$\frac{x}{c} - \frac{\sqrt{2} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}\right)b}{2c\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}\right)a}{\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}} - \frac{\sqrt{2} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}{\sqrt{-4ac+b^{2}}\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}} - \frac{\sqrt{2} \arctan\left(\frac{cx\sqrt{2}}{\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}}{2c\sqrt{-4ac+b^{2}}\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)b^{2}}}{2c\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}}{2c\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}\right)c}}\right)a}} - \frac{\sqrt{2} \operatorname{arctanh}\left$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \frac{x^9}{\left(c\,x^5 + b\,x^3 + a\,x\right)^2} \,\mathrm{d}x$$

Optimal(type 3, 120 leaves, 8 steps):

$$-\frac{bx^{2}}{2c(-4ac+b^{2})} + \frac{x^{4}(bx^{2}+2a)}{2(-4ac+b^{2})(cx^{4}+bx^{2}+a)} + \frac{b(-6ac+b^{2})\operatorname{arctanh}\left(\frac{2cx^{2}+b}{\sqrt{-4ac+b^{2}}}\right)}{2c^{2}(-4ac+b^{2})^{3/2}} + \frac{\ln(cx^{4}+bx^{2}+a)}{4c^{2}}$$

Result(type 3, 341 leaves):

$$\frac{b\left(3\,a\,c-b^{2}\right)x^{2}}{c^{2}\left(4\,a\,c-b^{2}\right)} + \frac{a\left(2\,a\,c-b^{2}\right)}{\left(4\,a\,c-b^{2}\right)c^{2}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)} + \frac{\ln\left(\left(4\,a\,c-b^{2}\right)c\left(c\,x^{4}+b\,x^{2}+a\right)\right)}{4\,c^{2}} - \frac{3\arctan\left(\frac{2\,c^{2}\left(4\,a\,c-b^{2}\right)x^{2}+\left(4\,a\,c-b^{2}\right)c\,b}{\sqrt{64\,a^{3}\,c^{5}-48\,a^{2}\,b^{2}\,c^{4}+12\,a\,b^{4}\,c^{3}-b^{6}\,c^{2}}}\right)b^{a}}{\sqrt{64\,a^{3}\,c^{5}-48\,a^{2}\,b^{2}\,c^{4}+12\,a\,b^{4}\,c^{3}-b^{6}\,c^{2}}}$$

$$+ \frac{\arctan\left(\frac{2\,c^{2}\left(4\,a\,c-b^{2}\right)x^{2}+\left(4\,a\,c-b^{2}\right)c\,b}{\sqrt{64\,a^{3}\,c^{5}-48\,a^{2}\,b^{2}\,c^{4}+12\,a\,b^{4}\,c^{3}-b^{6}\,c^{2}}}\right)b^{3}}{2\sqrt{64\,a^{3}\,c^{5}-48\,a^{2}\,b^{2}\,c^{4}+12\,a\,b^{4}\,c^{3}-b^{6}\,c^{2}}}$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{x^8}{\left(cx^5 + bx^3 + ax\right)^2} \, \mathrm{d}x$$

Optimal(type 3, 227 leaves, 6 steps):

$$-\frac{bx}{2c(-4ac+b^2)} + \frac{x^3(bx^2+2a)}{2(-4ac+b^2)(cx^4+bx^2+a)} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b-\sqrt{-4ac+b^2}}}\right) \left(b^2 - 6ac - \frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^3/^2(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b^2 - 6ac + \frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^3/^2(-4ac+b^2)\sqrt{b-\sqrt{-4ac+b^2}}} + \frac{\arctan\left(\frac{x\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{-4ac+b^2}}}\right) \left(b^2 - 6ac + \frac{b(-8ac+b^2)}{\sqrt{-4ac+b^2}}\right)\sqrt{2}}{4c^3/^2(-4ac+b^2)\sqrt{b+\sqrt{-4ac+b^2}}}$$

Result(type ?, 2157 leaves): Display of huge result suppressed!

Problem 28: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(cx^5 + bx^3 + ax\right)^2} \, \mathrm{d}x$$

Optimal(type 3, 260 leaves, 6 steps):

$$\frac{10\,a\,c-3\,b^{2}}{2\,a^{2}\left(-4\,a\,c+b^{2}\right)x} + \frac{b\,cx^{2}-2\,a\,c+b^{2}}{2\,a\left(-4\,a\,c+b^{2}\right)x\left(cx^{4}+b\,x^{2}+a\right)} - \frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b-\sqrt{-4\,a\,c+b^{2}}}}\right)\sqrt{c}\left(3\,b^{3}-16\,a\,b\,c+\left(-10\,a\,c+3\,b^{2}\right)\sqrt{-4\,a\,c+b^{2}}\right)\sqrt{2}}{4\,a^{2}\left(-4\,a\,c+b^{2}\right)^{3/2}\sqrt{b-\sqrt{-4\,a\,c+b^{2}}}} + \frac{\arctan\left(\frac{x\sqrt{2}\,\sqrt{c}}{\sqrt{b+\sqrt{-4\,a\,c+b^{2}}}}\right)\sqrt{c}\left(3\,b^{3}-16\,a\,b\,c-\left(-10\,a\,c+3\,b^{2}\right)\sqrt{-4\,a\,c+b^{2}}\right)\sqrt{2}}{4\,a^{2}\left(-4\,a\,c+b^{2}\right)^{3/2}\sqrt{b+\sqrt{-4\,a\,c+b^{2}}}}$$

Result(type ?, 2011 leaves): Display of huge result suppressed!

Problem 29: Result more than twice size of optimal antiderivative.

$$\left|\frac{1}{x\left(cx^{5}+bx^{3}+ax\right)^{2}} dx\right|$$

Optimal(type 3, 155 leaves, 9 steps):

$$\frac{3\,a\,c-b^2}{a^2\left(-4\,a\,c+b^2\right)x^2} + \frac{b\,cx^2 - 2\,a\,c+b^2}{2\,a\left(-4\,a\,c+b^2\right)x^2\left(cx^4 + b\,x^2 + a\right)} - \frac{\left(6\,a^2\,c^2 - 6\,a\,b^2\,c+b^4\right)\operatorname{arctanh}\left(\frac{2\,cx^2 + b}{\sqrt{-4\,a\,c+b^2}}\right)}{a^3\left(-4\,a\,c+b^2\right)^{3/2}} - \frac{2\,b\ln(x)}{a^3} + \frac{b\ln(cx^4 + b\,x^2 + a)}{2\,a^3}$$

Result(type 3, 568 leaves):

\_

$$\frac{c^{2}x^{2}}{a\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{cx^{2}b^{2}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} - \frac{3\,b\,c}{2\,a\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a^{2}\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a\left(cx^{4}+bx^{2}+a\right)\left(4\,a\,c-b^{2}\right)} + \frac{b^{3}}{2\,a\left(c$$

Problem 30: Result more than twice size of optimal antiderivative.

$$\frac{1}{x^2 \left(c x^5 + b x^3 + a x\right)^2} \, \mathrm{d}x$$

Optimal(type 3, 311 leaves, 7 steps):

$$\frac{14 a c - 5 b^{2}}{6 a^{2} (-4 a c + b^{2}) x^{3}} + \frac{b (-19 a c + 5 b^{2})}{2 a^{3} (-4 a c + b^{2}) x} + \frac{b c x^{2} - 2 a c + b^{2}}{2 a (-4 a c + b^{2}) x^{3} (c x^{4} + b x^{2} + a)}$$

$$+ \frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b - \sqrt{-4} a c + b^{2}}}\right) \sqrt{c} (5 b^{4} - 29 a b^{2} c + 28 a^{2} c^{2} + b (-19 a c + 5 b^{2}) \sqrt{-4 a c + b^{2}}) \sqrt{2}}{4 a^{3} (-4 a c + b^{2})^{3/2} \sqrt{b - \sqrt{-4} a c + b^{2}}}$$

$$- \frac{\arctan\left(\frac{x \sqrt{2} \sqrt{c}}{\sqrt{b + \sqrt{-4} a c + b^{2}}}\right) \sqrt{c} (5 b^{4} - 29 a b^{2} c + 28 a^{2} c^{2} - b (-19 a c + 5 b^{2}) \sqrt{-4 a c + b^{2}}) \sqrt{2}}{4 a^{3} (-4 a c + b^{2})^{3/2} \sqrt{b + \sqrt{-4} a c + b^{2}}}$$

Result(type ?, 2348 leaves): Display of huge result suppressed!

Problem 33: Result more than twice size of optimal antiderivative.

$$\int \frac{\sqrt{x}}{\left(cx^5 + bx^3 + ax\right)^3/2} \, \mathrm{d}x$$

Optimal(type 3, 87 leaves, 3 steps):

$$-\frac{\arctan\left(\frac{(b\,x^2+2\,a)\,\sqrt{x}}{2\,\sqrt{a}\,\sqrt{c\,x^5+b\,x^3+a\,x}}\right)}{2\,a^{3/2}}+\frac{(b\,c\,x^2-2\,a\,c+b^2)\,\sqrt{x}}{a\,(-4\,a\,c+b^2)\,\sqrt{c\,x^5+b\,x^3+a\,x}}$$

Result(type 3, 178 leaves):

$$-\frac{1}{2 a^{3/2} \sqrt{x} (c x^4 + b x^2 + a) (4 a c - b^2)} \left( \sqrt{x (c x^4 + b x^2 + a)} \left( 2 x^2 b c \sqrt{a} + 4 \ln \left( \frac{2 a + b x^2 + 2 \sqrt{a} \sqrt{c x^4 + b x^2 + a}}{x^2} \right) a c \sqrt{c x^4 + b x^2 + a} - \ln \left( \frac{2 a + b x^2 + 2 \sqrt{a} \sqrt{c x^4 + b x^2 + a}}{x^2} \right) b^2 \sqrt{c x^4 + b x^2 + a} - 4 a^{3/2} c + 2 b^2 \sqrt{a} \right) \right)$$

Problem 34: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{(cx^5 + bx^3 + ax)^{3/2} \sqrt{x}} \, \mathrm{d}x$$

Optimal(type 4, 454 leaves, 6 steps):

$$\frac{2(-3\,a\,c+b^2)\,x^{3/2}\,(cx^4+bx^2+a)\,\sqrt{c}}{a^2\,(-4\,a\,c+b^2)\,\sqrt{x}\,\sqrt{cx^5+bx^3+ax}} + \frac{b\,cx^2-2\,a\,c+b^2}{a\,(-4\,a\,c+b^2)\,\sqrt{x}\,\sqrt{cx^5+bx^3+ax}} - \frac{2\,(-3\,a\,c+b^2)\,\sqrt{cx^5+bx^3+ax}}{a^2\,(-4\,a\,c+b^2)\,x^{3/2}} - \frac{2\,c^{1/4}\,(-3\,a\,c+b^2)\,\sqrt{cx^5+bx^3+ax}}{\left(\sqrt{a}+x^2\sqrt{c}\right)\,\sqrt{cx}\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2} \operatorname{Elliptice}\left(\sin\left(2\,\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right), \frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\right)\,\sqrt{x}\,\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{a}+x^2\sqrt{c}\,)^2}} - \frac{c^{1/4}\,(-3\,a\,c+b^2)\,\sqrt{cx}\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2}{\cos\left(2\,\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)a^{7/4}\,(-4\,a\,c+b^2)\,\sqrt{cx^5+b\,x^3+ax}} + \frac{1}{2\,\cos\left(2\,\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)a^{7/4}\,(-4\,a\,c+b^2)\,\sqrt{cx^5+b\,x^3+ax}} \left(c^{1/4}\,\sqrt{\cos\left(2\,\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)^2} \operatorname{EllipticF}\left(\sin\left(2\,\arctan\left(\frac{c^{1/4}x}{a^{1/4}}\right)\right)\right), \frac{\sqrt{2-\frac{b}{\sqrt{a}\,\sqrt{c}}}}{2}\right)\left(\sqrt{a}+x^2\sqrt{c}\,)\,(2\,b^2-6\,a\,c+b\,\sqrt{a}\,\sqrt{c}\,)\,\sqrt{x}\,\sqrt{\frac{cx^4+bx^2+a}{(\sqrt{a}+x^2\sqrt{c}\,)^2}}}\right)$$

Result(type 4, 1135 leaves):

$$- \left(\sqrt{x(cx^3 + bx^2 + a)} \left( 12\sqrt{-4ac + b^2} \sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}} x^4 ac^2 - 4\sqrt{-4ac + b^2} \sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}} x^4 b^2 c + 12\sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}} x^4 bbc^2 - 4\sqrt{\frac{-b + \sqrt{-4ac + b^2}}{a}} x^4 b^5 c + c\sqrt{\frac{-2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^4 b^5 c + c\sqrt{\frac{-2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^4 b^2 c + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}{a}}}{2} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^4 b^2 c + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^4 b^2 c + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^4 b^2 c + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - 2ac + b^2)}}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}}{a}} \sqrt{\frac{x^2\sqrt{-4ac + b^2} + bx^2 + 2a}{a}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2 - 2a)}}}} x^2 + \frac{12\sqrt{-\frac{2(x^2\sqrt{-4ac + b^2} - bx^2$$

$$\frac{\sqrt{2}\sqrt{\frac{b\sqrt{-4\,a\,c+b^2}-2\,a\,c+b^2}}{2}}{2}\right)x\,a\,b^2\,c+14\sqrt{-4\,a\,c+b^2}\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}x^2\,a\,b\,c-4\sqrt{-4\,a\,c+b^2}\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}x^2\,b^3$$

$$+14\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}x^2\,a\,b^2\,c-4\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}x^2\,b^4+8\sqrt{-4\,a\,c+b^2}\sqrt{\frac{-b+\sqrt{-4\,a\,c+b^2}}{a}}a^2\,c$$

$$-2\sqrt{-4ac+b^{2}}\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}} ab^{2}+8\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}} a^{2}bc-2\sqrt{\frac{-b+\sqrt{-4ac+b^{2}}}{a}} ab^{3} \right) \right) / \left( 2x^{3/2}(cx^{4}+bx^{2}+a)(4ac^{2}+bx^{2}+bx^{2}+a)(4ac^{2}+bx^{2}+bx^{2}+a)(4ac^{2}+bx^{2}+bx^{2}+bx^{2}+bx^{2}+bx^{2}+a)(4ac^{2}+bx^{2}+b$$

Problem 36: Unable to integrate problem.

$$\frac{x(ex^2+d)}{\sqrt{cx^5+bx^3+ax}} \, \mathrm{d}x$$

Optimal(type 6, 239 leaves, 7 steps):

$$\frac{2 \, dx^2 \, AppellFI\left(\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 \, x^2 \, c}{b - \sqrt{-4 \, a \, c + b^2}}, -\frac{2 \, x^2 \, c}{b + \sqrt{-4 \, a \, c + b^2}}\right) \sqrt{1 + \frac{2 \, x^2 \, c}{b - \sqrt{-4 \, a \, c + b^2}}} \sqrt{1 + \frac{2 \, x^2 \, c}{b + \sqrt{-4 \, a \, c + b^2}}}{3 \, \sqrt{c \, x^5 + b \, x^3 + a \, x}}} + \frac{2 \, e \, x^4 \, AppellFI\left(\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 \, x^2 \, c}{b - \sqrt{-4 \, a \, c + b^2}}, -\frac{2 \, x^2 \, c}{b + \sqrt{-4 \, a \, c + b^2}}\right) \sqrt{1 + \frac{2 \, x^2 \, c}{b - \sqrt{-4 \, a \, c + b^2}}}}{\sqrt{1 + \frac{2 \, x^2 \, c}{b + \sqrt{-4 \, a \, c + b^2}}}} \sqrt{1 + \frac{2 \, x^2 \, c}{b + \sqrt{-4 \, a \, c + b^2}}}}{7 \, \sqrt{c \, x^5 + b \, x^3 + a \, x}}}$$

Result(type 8, 27 leaves):

$$\int \frac{x (ex^2 + d)}{\sqrt{cx^5 + bx^3 + ax}} \, \mathrm{d}x$$

Summary of Integration Test Results

40 integration problems



- A 27 optimal antiderivatives
  B 12 more than twice size of optimal antiderivatives
  C 0 unnecessarily complex antiderivatives
  D 1 unable to integrate problems
  E 0 integration timeouts